Master's Thesis

Epidemic models for financial effects influenced by US subprime mortgage crisis in 2008 on Korean corporations by classifying industries

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Abstract

In modern society, financial industry and human-being establish a truly inseparable relationship. Moreover, financial industry becomes inter-correlated market following the advance of globalization. Thus, they interact both directly and indirectly with each other. Therefore, if an outbreak of financial crisis occurs in one sector, one can anticipate that the crisis effect will diffuse to other financial sectors similar with contagious disease.

This research analyzes and does modeling of diffusion of financial crisis from one financial sector to other sectors by using epidemic models. This work mainly focuses on the impact and contagious phenomenon of US financial shock developed in 2008 on Korean corporations enlisted in KOSPI & KOSDAQ index. And then, more precisely, by classifying industries as several groups, it analyzes how financial crisis influences on each group.

The SIR model (Susceptible-Infected-Removed model) is set and the aspect of the model is compared with the result obtained from a quantitative indicator, EDF model (Expected Default Frequency model).

Throughout this research, the validity of using epidemic models is discussed how proper it is, to estimate the diffusion of financial crisis to another financial sector or country, and furthermore, to each industry group within one financial sector.

In this research, the fundamental data from KRX and FSS is used for EDF model. It includes stock price, total market value, and current liabilities of each corporation from Jan. 2007 to Dec. 2010. From this data, the contagious features started from US financial crisis are observed – within infected corporations, EDF value is increase or maintains sustained level right after US crisis.

With respect to the average EDF value and DD value of each group, parameters for deterministic epidemic model, including contact rate and recovery rate, are presumed.

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I. Introduction

Because the global financial markets are organically linked, the financial industry is a highly cross-correlated market. Thus, a change in one financial market can directly or indirectly affect other markets. That is to say, if a financial crisis occurs in one country, we can conclude that the crisis will easily affect other financial markets, countries, and international financial markets. This phenomenon can be regarded as an epidemic phenomenon that occurs between financial markets. In other words, the financial crisis of a financial market can be transmitted to other financial markets, and this effect will spread like an infectious disease again in an infected financial market.

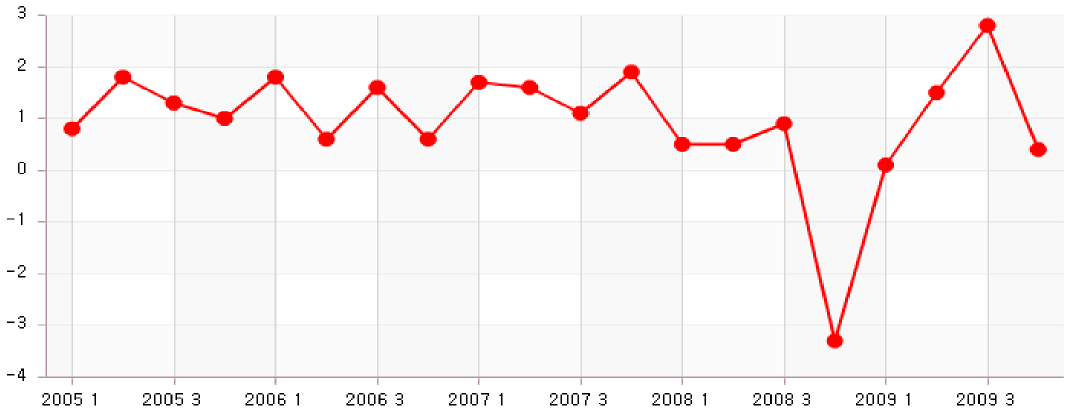
In this paper, we will deal with US Subprime Mortgage crisis, a representative example of financial crisis. Before this financial crisis, there were many financial crises such as the 1997 Asian crisis including Indonesia, Malaysia, the Philippines, South Korea and Thailand, and the 1998 Russian crisis. However, since the US Subprime Mortgage crisis has occurred most recently and has had a huge impact on many financial markets around the world, the proliferation of the financial crisis can also be easily identified.

In the early 2000s, the US Federal Reserve continued to reduce federal funds rates. With the policy of freezing 1% interest rates until June 2004, subprime mortgage lending for lenders with low credit ratings began to increase. In addition, as the lending conditions have eased and house prices have risen, lower interest rates have become a driving force for subprime mortgage lending to explode.

However, in 2006, as the Federal Reserve implemented a policy of gradually raising interest rates, the interest burden on those who received subprime mortgage loans increased. Moreover, as the housing market sharply declined due to rising interest rates, lending companies began to collapse in 2007.

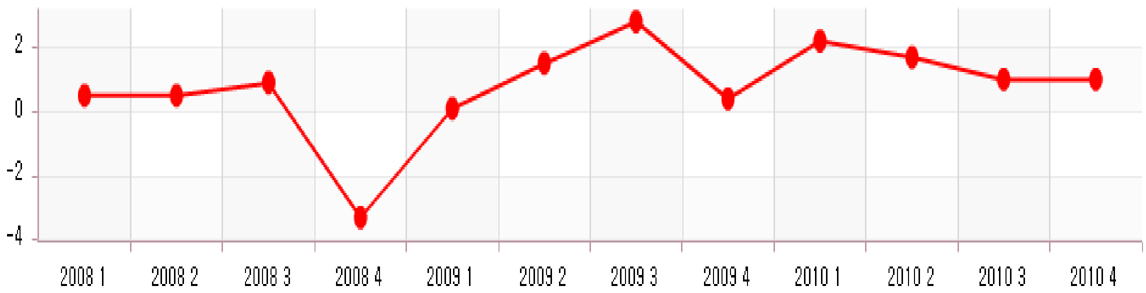
The financial crisis, which began in April 2007 when New Century Financial Corporation, a subprime mortgage lender, filed for bankruptcy, was boosted by Lehman Brothers Holdings Inc. filing for bankruptcy in September 2008. These US subprime mortgage crises quickly spread to the worldwide real economy. [Figure 1] is a graph of Korea GDP from the first quarter of 2005 to the fourth quarter of 2009. As such, the US subprime mortgage crisis has spread rapidly to the domestic economy.

[Figure 1] GDP from the first quarter of 2005 to the fourth quarter of 2009[1]



In addition, US Subprime mortgage crisis has also directly or indirectly affected the domestic real economy. Since Korea is a country with a high dependence on exports, it suffered a negative impact on the real economy, such as a decline in exports due to the international economic recession and a decrease in consumption due to the won depreciation and a sharp fall in share prices. During this period, the Korean economy grew by -3.4% in the last quarter of 2008 compared to the previous year, with manufacturing and construction industries, mainstay industries, growing by -9.1% and -6.3%, respectively.

[Figure 2] Economic growth rate[2]



The KOSPI index also plummeted shortly after Lehman Brothers Holdings Inc. filed for bankruptcy, falling below the 1000-point mark in October 2008. The US subprime mortgage crisis has affected not only the financial market but also the Korean economy as a whole.

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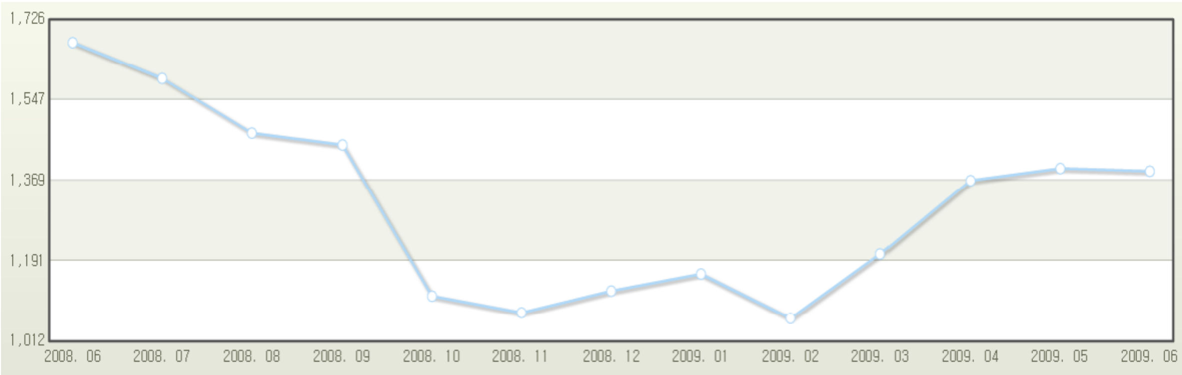
[1] Economic Statistics System of Bank of Korea (http://ecos.bok.or.kr/)

[2] Economic Statistics System of Bank of Korea (http://ecos.bok.or.kr/)

[Figure 3] KOSPI Index[3]



Right after US subprime mortgage crisis, there has been a need to find and develop ways to recognize and prevent financial crises. If we can detect the financial crisis in advance and design a way to cope with it effectively, we will be able to prevent the excessive swings in the financial market. Therefore, the purpose of this study is to determine if it is possible to use the epidemic model to predict and cope with the financial crisis. We also want to identify the epidemic models that are appropriate for the proliferation of financial crises in each industry by classifying industries.

In order to apply the epidemic model, we need a model that can calculate the bankruptcy probability numerically because the parameter must be estimated from the possibility of bankruptcy of each company before and after the financial crisis. The bankruptcy prediction model can be classified into a model using financial data disclosed by the company and a model using market information. Since the bankruptcy prediction model using financial data has a limitation that it cannot overcome the lagging phenomenon[4], we use the model based on market information. In this paper, we use the KMV EDF model that predicts the default probability of firms using stock price information to quantify the default probability of an individual firm and estimate the parameters for the epidemic model based on the KMV EDF model.

Section 1 describes the background and existing research of this study. Section 2 describes the theoretical model for the epidemic and EDF models. In Section 3, modeling, analysis and results are presented. In Section 4, conclusions are drawn and limitations of this study are described.

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3 Korean Statistical Information Service(KOSIS) (http://kosis.kr/)

4 Since the timing of financial data is past, it is difficult to cope with the rapidly changing economic situation when it is used to predict the degree of credit risk.

1.1. Existing research

1.1.1. Definition of spreading the financial crisis

M. Pericoli and M. Sbracia (2003) defined the spread of the financial crisis as the following five;

1) Contagion is a significant increase in the probability of a crisis in one country, conditional on a crisis occurring in another country.

2) Contagion occurs when volatility of asset prices spills over from the crisis country to other countries.

3) Contagion occurs when cross-country comovements of asset prices cannot be explained by fundamentals.

4) Contagion is a significant increase in comovements of prices and quantities across markets, conditional on a crisis occurring in one market or group of markets.

5) Contagion occurs when the transmission channel intensifies or, more generally, changes after a shock in one market.

1.1.2. KMV EDF model

EDF model proposed by KMV corporation is a default prediction model using stock price information. It is the most used bankruptcy prediction model in domestic and foreign banks.

McQuown, J. A. (1993) applied EDF to US firms and found that default risk measurement using stock price information is more accurate than bankruptcy risk measurement using existing accounting data.

Donggeol, L., & Seijin, K. (2001) suggested that EDF is more useful for predicting bankruptcy than credit rating, and can identify changes in corporate credit risk. Chanpyo, K., & Wanho, J. (2002) have empirically verified that the use of the EDF model results in better bankruptcy prediction results for domestic market by comparing the predictions of the EDF model and other bankruptcy forecasting models.

Nayoung, K. (2003) classified the default risk of domestic companies into 16 industries under the special circumstances of IMF and showed that EDF could be used as a leading indicator in some industries. Jieun, H. (2009) analyzed the EDF from 2000 to 2008 of manufacturing, wholesale and retail, service, and construction industries and showed that the EDF model was useful as an early warning. Dongpil, S. (2010) analyzed the business groups classified by the stock market and the size of firms, by applying the optimal weighted ratio of non-current debt, and showed that EDF results for each industry preceded the economy.

1.1.3. Theoretical models for contagion of financial crises

In order to explain the contagious spread of the financial crisis, there have been various studies using financial linkages (F. Allen & D. Gale, 2000) and information asymmetry (G. Calvo & E. Mendoza, 2000). A. G. Haldane (2008) argued that because of the inherent complexity and inter-connectivity of the financial system, the methods used in the fields of ecology, epidemiology, biology, and engineering should be applied to explain the contagious spread of the financial crisis.

Thus, there have been attempts to model financial contagion using numerical simulations. For example, Gai, P. and Kapadia, S. (2010) studied a percolation-type process on a weighted network of banks as a model of contagion. Bond percolation processes are equivalent to susceptible–infected–recovered (SIR) epidemics given appropriate model specification (Newman, E. J. M., 2010). May, R. and Arinaminpathy, N. (2010) pursued recent advances in the area of complex ecological systems in a study that is similar in spirit to that of Gai, P. and Kapadia, N. (2010), though they employed a mean field approximation rather than resorting to simulations. Amini, *et al*. (2013) analyzed distress propagation in a network of banks, via a cascading process, and derived the asymptotic magnitude of contagion. Caporale, *et al*. (2009) made use of agent-based simulation models to investigate the dynamics of financial contagion.

Demiris, et al. (2012) have proposed a framework for modelling financial contagion that is based on SIR model from epidemic theory.

II. Theoretical & Mathematical models

2.1. Epidemic model

The epidemic model shows the aspects of the diffusion or spread of disease caused by toxic substances of a particular pathogen or pathogen, itself. In the case of infectious diseases, it can be called a disease spread model. The first reason for constructing the epidemic model is to obtain information on the characteristics of disease such as infection duration, propagation speed, incubation period, immunity after infection. The second reason is to prepare preventive measures against the characteristics of disease.

The states of SIR model are classified into three categories: S (Susceptible) - I (Infected) - R (Removed). Here, S (susceptible) refers to a target group that is susceptible to infection in the entire population. Next, the infected population from the susceptible population becomes I (infected), and the population of the infected population that has died or recovered from the infected population is represented by R (removed). SIR model is a model showing the spread of disease in the process, from S to I and from I to R, as described above. SIR model was proposed by W. O. Kermack and A. G. McKendrick (1927) to predict the prevalence and spread of disease epidemics.

<SIR Model>

Using a fixed population, N = S(t) + I(t) + R(t), Kermack and McKendrick derived the following equations:

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I \qquad \text{where, } \beta : \text{Contact rate}$$

$$\frac{dR}{dt} = \gamma I \qquad \gamma : \text{Recovery rate}$$

The states of SEIR model are represented by S (Susceptible) - E (Exposed) - I (Infected) - R (Removed). As in SIR model, S (susceptible) refers to a susceptible person to infection within the entire population. SEIR model differs from SIR model in that it has a latency period. The infected person becomes E (exposed) state, and the person in 'E' state transfers to I (infected) state when symptoms develop after the incubation period. A person who died or recovered from 'I' state transitions to R (removed). SEIR model thus proceeds from S to E, from E to I, and from I to R.

<SEIR Model>

Using a fixed population, N = S(t) + E(t) + I(t) + R(t), with following equations:

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dE}{dt} = \beta SI - \sigma E \qquad \text{where, } \beta : \text{Contact rate}$$

$$\frac{dI}{dt} = \sigma E - \gamma I \qquad \sigma : \text{Incubation rate}$$

$$\frac{dR}{dt} = \gamma I \qquad \gamma : \text{Recovery rate}$$

2.2. Expected Default Frequency (EDF) model

2.2.1. KMV EDF model

KMV EDF model is a model approach to systematic risk[5] developed by KMV Corporation, which was established in 1989 with the aim of measuring credit risk. The model predicts the bankruptcy of firms based on information in the securities market. The expected default frequency (EDF), which can

[5] It is a method that stipulates that 'Default' occurs when the debt of a company exceeds the market value of asset. In stock market, the probability of default is determined by stock price, which includes information that can be used to estimate the default probability of an individual company, such as the financial structure of a company. This is called the 'model approach to systematic risk' because it is determined by the volatility of asset value and the financial structure of company.

represent the default probability of an individual firm as a value between 0 and 1, is used.

KMV EDF model is based on Merton model (Merton, R.C., 1974), which is influenced by Black-Scholes' option pricing model (Black, F. & Scholes, M., 1973). Merton model considers corporate equity, owned by shareholders, as an option to utilize Black-Scholes' option pricing model, which is an option valuation model based on market information, to measure the firm's asset value. The probability of default is calculated by substituting it into the put-call parity of the option pricing model. Black-Scholes-Merton model is based on the assumption that the asset value of an individual firm follows a geometric Brownian motion.

$$dS = \mu S dt + \sigma S dW \tag{1}$$

where μ is the stock's expected rate of return and σ is the volatility of the stock price.

KMV EDF model estimates the future asset value of an individual firm by using the estimated asset value and volatility of asset return from the stock price information. And it assumes that default occurs when the asset value falls below the default point (DPT). In order to calculate DPT, it uses a linear combination of short and long-term debt as in (2). The adjustment factor α is a value between 0 and 1. In this paper, we use 0.5 as α which is the most commonly used.

$$\mathrm{DPT} = \mathrm{STD} + \alpha \times \mathrm{LTD} \tag{2}$$

That is, DPT is the book value of the liability that must be repaid within one year.

To obtain the expected default rate (EDF), determine the market value and volatility of the firm and then calculate the Distance to Default (DD)[6] between DPT and the market value. Finally, convert Distance to Default to EDF.

[6] Standard deviation of the distance between the average of the asset value and the default point

2.2.1.1 Estimate the Asset value and Volatility of the asset value

The value of assets (V_A) and volatility of the asset value (σ_A) of an individual company are computed by using the equity value (V_E) of the company. These two variables can be estimated simultaneously by calculating two simultaneous equations introduced in Merton (1974) model.

First, by using Black-Scholes option pricing model (3), we can see the relationship between the value of the equity (V_E) and asset value (V_A) at the time of debt maturity (T).

$$V_E = V_A N(d_1) - V_D e^{-rT} N(d_2), \tag{3}$$

where $d_1 = \dfrac{\ln\left({V_A}/{V_D}\right) + \left(r + {\sigma^2{}_A}/{2}\right)T}{\sigma_A \sqrt{T}}$

and $d_2 = d_1 - \sigma_A \sqrt{T}$

Followings are definitions of each parameter;

V_A : Asset value

V_E : Value of equity

V_D : Book value of liability

r : Risk-free interest rate

σ_A : Volatility of asset value

T : Debt maturity

N : Cumulative distribution function of standard normal distribution

And, the volatility of value of equity (σ_E) can be obtained by multiplying volatility of the asset value (σ_A) with the asset value (V_A) after dividing by the value of equity (V_E). Thus, the relation between σ_E and σ_A can be expressed by (4).

$$\sigma_E = \frac{V_A}{V_E} N(d_1)\sigma_A, \tag{4}$$

Then, (3) and (4) are taken together to estimate an approximation of the asset value (V_A) and the volatility (σ_A) of the asset value of the individual firm. 'Newton-Raphson method'[7], one of the iteration methods, is used as an estimation method.

The following equation (5) is a real-valued function f derived by (3) and (4).

$$f(V_A, \sigma_A) = \begin{bmatrix} f_1(V_A, \sigma_A) \\ f_2(V_A, \sigma_A) \end{bmatrix} = \begin{bmatrix} V_A N(d_1) - V_D e^{-rT} N(d_2) - V_E \\ \frac{V_A}{V_E} N(d_1)\sigma_A - \sigma_E \end{bmatrix} = 0 \tag{5}$$

Jacobian matrix is introduced to estimate the solution of the above equation by Newton-Raphson method. (Stoer & Bulirch, 1992) Following partial derivatives are refer to the doctoral dissertation of Dongpil, S. (2016, p. 27).

$$\mathrm{Df}(V_A, \sigma_A) = \begin{bmatrix} \frac{\partial f_1}{\partial V_A} & \frac{\partial f_1}{\partial \sigma_A} \\ \frac{\partial f_2}{\partial V_A} & \frac{\partial f_2}{\partial \sigma_A} \end{bmatrix} \tag{6}$$

$$\frac{\partial f_1}{\partial V_A} = N(d_1) + \frac{V_A P(d_1) - V_D e^{-rT} P(d_2)}{V_D \sigma_A \sqrt{T}} \tag{7}$$

$$\frac{\partial f_1}{\partial \sigma_A} = \frac{-V_A P(d_1)\cdot d_2 + V_D e^{-rT} P(d_2)\cdot d_1}{V_D \sigma_A \sqrt{T}} \tag{8}$$

$$\frac{\partial f_2}{\partial V_A} = \frac{N(d_1)\sigma_A \sqrt{T} + P(d_1)}{V_E \sqrt{T}} \tag{9}$$

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[7] To estimate the root of a real-valued function f, we obtain a better estimate x_1 by using the initial value x_0 and the derivative f ' of f. And repeat this process to find the optimal estimate for the root of ta real-valued function f.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\frac{\partial f_2}{\partial \sigma_A} = \frac{V_A[N(d_1) - d_2 \cdot P(d_1)]}{V_E} \tag{10}$$

(11) is the solution of Newton-Raphson method by using Jacobian matrix (6).

$$(V_A^{n+1}, \sigma_A^{n+1}) = (V_A^n, \sigma_A^n) - Df^{-1}(V_A^n, \sigma_A^n) \times f(V_A^n, \sigma_A^n), n = 0,1,2 \ldots \tag{11}$$

When V_A^{n+1}, σ_A^{n+1} obtained by repeated calculation (11) converge to zero with less than a certain level of error, $V_A^{n+1}\ and\ \sigma_A^{n+1}$ are used as the estimate of the asset value (V_A) and the volatility of the asset value (σ_A).

2.2.1.2 Estimate the Distance to Default (DD) & EDF value

Assuming that the distribution of asset values is a normal distribution based on the assumption of Merton (1974), we use (12) to calculate Distance to Default. Assuming that the distribution of asset values is normal distribution, the larger the Distance to Default is, the lower probability of default is.

$$DD = d_2 = d_1 - \sigma_A\sqrt{T} = \frac{\ln\left({V_A}/{V_D}\right) + \left(r - {\sigma^2{}_A}/{2}\right)T}{\sigma_A\sqrt{T}} \tag{12}$$

According to Crosbie and Bohn (2003), the theoretical expected default probability (EDF) of an individual firm is $N(-d_2)$. (Bohn & Crosbie, 2003)

III. Application

3.1. Data Collection & Variable Setting and Defining Status

3.1.1. Data Collection

Data for the last 252 business days are required to obtain the EDF value. The data used in this study are those of listed companies on the KOSPI market and having stock price information from January 1, 2007 to December 31, 2010. The financial sector (KSIC codes 64, 65, 66) which is not suitable to apply EDF model and the data of the debt data missing companies and listed companies after January 1, 2007 are excluded. In order to classify industries, KSIC Korea Standard Industrial Classification[8], published by the National Statistical Office, was used as a standard.

[Table 1] Industry Classification

Classification	KSIC Code
Heavy	17, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31
Light	1, 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 18, 22, 32, 33
Service	35, 36, 37, 38, 39, 49, 50, 51, 52, 55, 56, 58, 59, 60, 61, 62, 63, 70, 71, 72, 73, 74, 75, 76, 84, 85, 86, 90, 94, 95, 96
Construction	41, 42, 68
W&R trade	45, 46, 47

The status of the companies that were finally refined according to the industrial classification and the status of the bankrupt companies by each industry are as follows.

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[8] Korean Statistical Information Service (KOSIS)

(http://kssc.kostat.go.kr/ksscNew_web/kssc/common/ClassificationContent.do?gubun=1&strCategoryNameCode=001&categoryMenu=007)

[Table 2] Number of Corporations for each industry

	Whole	Heavy	Light	Service	Construction	W&R trade
Total	555	302	89	100	19	45
Defaulted	23	14	2	4	3	0

In the case of wholesale and retail trade, since no default occurred during the period, we will exclude it from simulation using SIR and SEIR models.

In this study, bankruptcy is defined as the cancellation of listing under the KOSPI market because of the use of financial information of the company listed on the KOSPI market. Among the companies used in this study, companies that meet the requirements for the delisting of Korea Exchange and the reason for the delisting are as follows.

[Table 3] Information on Delisted Corporations

Name	Industry	ISIN No.	Date of Abolishment	Reason for Delisting
유리ES	Heavy	A007050	2009.4.15	Going to the final bankruptcy
세신	Heavy	A004230	2009.4.30	Rejection of the audit opinion
BHK	Heavy	A003990	2009.4.30	Rejection of the audit opinion
기린	Light	A006070	2009.5.13	Rejection of the audit opinion
C&우방	Construction	A013200	2009.5.13	Rejection of the audit opinion
C&중공업	Heavy	A008400	2009.5.13	Rejection of the audit opinion
KMH	Service	A009690	2009.5.13	Rejection of the audit opinion

KTF	Service	A032390	2009.6.23	Occurrence of dissolution reason
현대Autonet	Heavy	A042100	2009.7.16	Occurrence of dissolution reason
FNC코오롱	Light	A001370	2009.8.17	Occurrence of dissolution reason
남한제지	Heavy	A001950	2009.10.15	Encroachment of more than 50/100 of total capital (continue to 2 years)
LG데이콤	Service	A015940	2010.1.15	Occurrence of dissolution reason
한국기술산업	Heavy	A008320	2010.3.11	Rejection of the audit opinion
서광건설산업	Construction	A001600	2010.4.15	Encroachment of the total capital
조인에너지	Heavy	A004820	2010.4.15	Encroachment of the total capital
고제	Heavy	A002540	2010.4.23	Rejection of the audit opinion
제로원인터랙티브	Service	A069470	2010.5.3	Rejection of the audit opinion
성원건설	Construction	A012090	2010.5.3	Rejection of the audit opinion
유성TSI	Heavy	A024870	2010.5.3	Rejection of the audit opinion
태창기업	Heavy	A007490	2010.5.13	Rejection of the audit opinion
현대금속	Heavy	A018410	2010.5.13	Rejection of the audit opinion
KCO에너지	Heavy	A011400	2010.5.20	Going to the final bankruptcy
한국고덴시	Heavy	A027840	2010.7.14	Occurrence of dissolution reason

The stock price information used in this study is obtained through Korea Exchange (http://www.krx.co.kr), and the debt information is obtained from the electronic disclosure system (http://dart.fss.or.kr) managed by Financial Supervisory Service. Listed companies are obliged to disclose debt information every quarter. Companies that did not observe disclosure obligations and did not have debt information for that period were excluded from this study. In addition, since the published debt information is quarterly data, the monthly debt information of firms is interpolated by linear interpolation[9]. The market interest rate is based on 4.1.2 market interest rate (month, quarter) - government bond (1 year) data of Economic Statistics System of Bank of Korea (https://ecos.bok.or.kr).

3.1.2. Variable Setting and Defining Status

3.1.2.1. Definition of variables for EDF calculation

It is very important to set the appropriate time unit since the stock price information used to estimate the default probability of an individual firm using EDF model is determined from highly mutable variables over time.

Through data collection, data related to stock prices such as the closing price and market capitalization were collected on a daily basis, but debt data could only be collected monthly. In addition, if the time unit is too short, it is very difficult to calculate EDF. Therefore, in this study, one month is set as the reference time unit.

The value of equity (V_E)of a firm is defined as the daily closing price of common stock, and the volatility of share price (σ_E) is calculated by Historical Volatility Calculation[10]. In addition, the initial value (V_A^0) of the asset value is the sum of the market value of the stock price and the debt book value, and the initial volatility of equity (σ_E^0) is used as the initial value of the variability of the asset value (σ_A^0). (Dongpil, S., 2010)

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[9] An interpolation method which estimates the value located between two given values of both end points in proportion to the linear distance.

[10] Historical volatility is a statistical measure of the variance of returns on securities or market indices over a given period. Volatility is calculated by following equation;

$$\text{Volatility} = \sqrt{\frac{\sum(R_t - R_m)^2}{T-1}}, R_t = \frac{P_t - P_{t-1}}{P_{t-1}}, R_m = \sum\frac{R_t}{T}, when\ P_t\ is\ the\ stock\ price\ at\ time\ T$$

That is, we can use the closing price data for the previous 252 business days to show the degree of fluctuation of stock price return.

3.1.2.2. Definition of variables for Epidemic model

In order to properly link EDF model with the epidemic model, the time of onset of the epidemic(shock) must be established. It also needs to define the states (S (Susceptible), E (Exposed), I (Infected), R (Removed) and D (Dead)) required to design the epidemic model. Finally, it is necessary to estimate the parameters used in the transition from the current state to the next state.

US subprime mortgage crisis that occurred in 2008 presents several 'Shock' of various magnitudes. In this study, September 2008, which is the time of filing for bankruptcy of Lehman Brothers Holdings Inc., which had the greatest impact on KOSPI index decline, is defined as 'Shock'. By analyzing EDF trends starting from September 2008, define each state of the epidemic model. In other words, the rapid change in EDF that occurred before August 2008 is not considered to be an infectious disease. In addition, once a company is infected and then recovered, it is not infected again.

Followings are definition of transitions using for SIR & SEIR models;

[Table 4] Definition of transitions between two states for SIR model

S ⇒ **I**	1) EDF value increased more than 2 times compared to last month 2) EDF value increased by more than 1.5 times and less than 2 times & did not decrease below the previous value during the first quarter
I ⇒ **R**	EDF value is reduced (recovered) below the value before the infection
I ⇒ **D**	EDF value did not recover below the pre-infection value by December 2010

[Table 5] Definition of transitions between two states for SEIR model

S ⇒ **E**	1) EDF value increased more than 2 times compared to last month 2) EDF value increased by more than 1.5 times and less than 2 times & did not decrease below the previous value during the first quarter
E ⇒ **I**	EDF value did not decrease to the previous value during next 2 quarters
I ⇒ **R**	EDF value is reduced (recovered) below the value before the exposure
I ⇒ **D**	EDF value did not recover below the pre-exposure value by December 2010

3.2. Analysis & Results

The following table shows the Descriptive statistics related to EDF of 555 companies obtained by the method described in 2.2.[11]

[Table 6] Descriptive Statistics about EDF for 555 Sample Corp.

Year	EDF			
	Average	Max	Min	STDEV
2008	0.0371662684051298	1.0000000000000000	8.70460141898157E-81	0.103401484898721
2009	0.0484795706434768	1.0000000000000000	9.35913430398396E-56	0.0952544357152043
2010	0.0239803870189875	1.0000000000000000	3.21987987701141E-71	0.109749016723187

Next, the parameters for determining the state transition in SIR model and SEIR model are determined. In this study, since the newly listed companies are excluded from the study, the total number of companies in whole states is always constant. SIR model and SEIR model are summarized as follows.

< SIR Model >

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \delta I \qquad \text{where, } \beta : \text{Contact rate}$$

$$\frac{dR}{dt} = \gamma I \qquad \gamma : \text{Recovery rate}$$

$$\frac{dD}{dt} = \delta I \qquad \delta : \text{Morbidity}$$

$$S + I + R + D = C(\text{Constant})$$

[11] In the appendix, Descriptive statistics for each industry classification can be found.

Here, β (Contact rate) is the ratio of cumulative infected companies (I) among all companies. Also, γ (recovery rate) is the reciprocal of the average recovery period taken by the infected companies (I), and δ (Morbidity) is the ratio of companies that have not recovered until December 2010 among infected companies (I).

The following table shows the estimated parameter values for SIR model for the entire company and classified industries.

[Table 7] Parameters for SIR Model

	Contact rate (β)	**Recovery rate (γ)**	**Morbidity (δ)**
Whole	0.00167518870221573	0.0935969526573553	0.00640490114174325
Heavy	0.00300425419937722	0.087848669445335	0.00651720542231491
Light	0.0102259815679838	0.0924657534246575	0.00661375661375661
Service	0.0095	0.0893697083725306	0.00639097744360902
Construction	0.0470914127423823	0.0944444444444444	0.00840336134453781
W&R trade	0.020740741	0.08203125	0.00510204081632653

< SEIR Model >

$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dE}{dt} = \beta SI - \sigma E$$ where, β : Contact rate

$$\frac{dI}{dt} = \sigma E - \gamma I - \delta I$$ σ : Incubation rate

$$\frac{dR}{dt} = \gamma I$$ γ : Recovery rate

$$\frac{dD}{dt} = \delta I$$ δ : Morbidity

$$S + E + I + R + D = C(\text{Constant})$$

Here, β (Contact rate) is the ratio of cumulative infected companies (I) among all companies. σ (Incubation rate) is the ratio of the cumulative companies transferred to infected state (I) among the exposed companies (E). Also, γ (recovery rate) is the reciprocal of the average recovery period taken by the infected (I) companies after transition from exposed state (E), and δ (Morbidity) is the ratio of companies that have not recovered until December 2010 among infected companies (I).

The following table shows the estimated parameter values for SEIR model for the entire company and classified industries.

[Table 8] Parameters for SEIR Model

	Contact rate (β)	Incubation rate(σ)	Recovery rate (γ)	Morbidity (δ)
Whole	0.00168492817141466	0.940269749518304	0.128964059196617	0.00695257611241218
Heavy	0.00303714749353099	0.938628158844765	0.124282982791587	0.00714285714285714
Light	0.0103522282540083	0.951219512195122	0.138790035587189	0.00686813186813187
Service	0.0099	0.939393939393939	0.136363636363636	0.00691244239631336
Construction	0.0470914127423823	1	0.150442477876106	0.00840336134453781
W&R trade	0.0217283950617284	0.90909090909090909	0.124223602484472	0.00535714285714286

The initial values for SIR and SEIR models are as follows. The initial values are the number of companies that meet the criteria of 'I' and 'E' state in September 2008, the time of onset of the epidemic, and all other companies are in 'S' state.

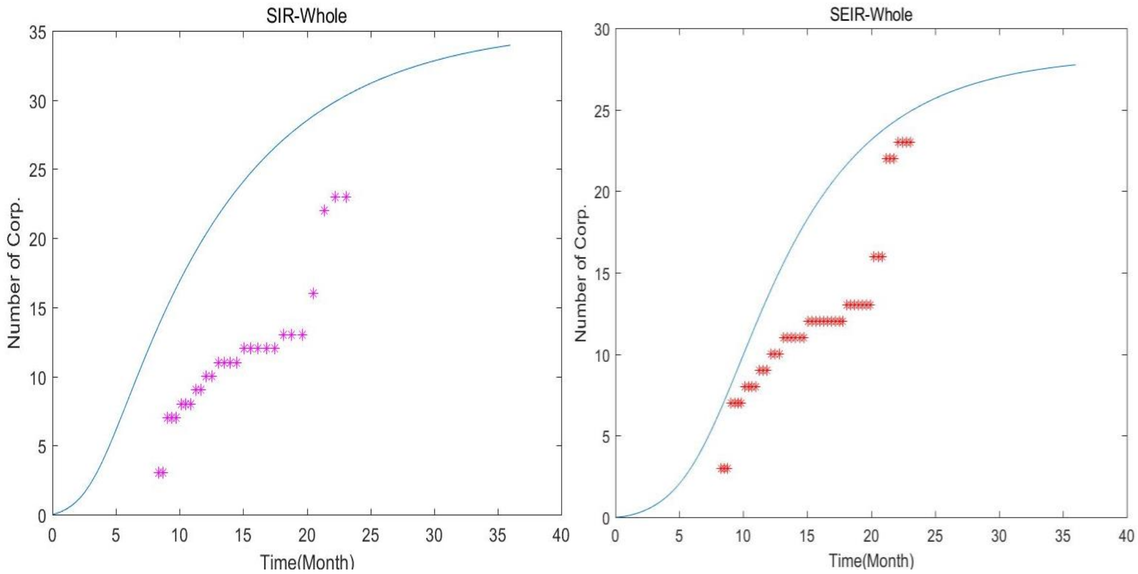
[Table 9] Initial Value for Epidemic Models

	SIR				SEIR				
	S	**I**	**R**	**D**	**S**	**E**	**I**	**R**	**D**
Whole	518	37	0	0	518	37	0	0	0
Heavy	284	18	0	0	284	18	0	0	0
Light	82	7	0	0	82	7	0	0	0
Service	94	6	0	0	94	6	0	0	0
Construction	15	4	0	0	15	4	0	0	0
W&R trade	43	2	0	0	43	2	0	0	0

3.2.1. Simulations for overall Corporations

[Figure 4] shows the 'D (Dead)' state graph of the overall company simulation using Matlab's ODE45 function. The '*' on the graph is the graph showing the actual default companies. Since the last bankruptcy occurred in July 2010, it is necessary to check the 'D (Dead)' state after 23 months based on August 2008, when the financial crisis occurred.

[Figure 4] Simulations for Whole Corporations by SIR & SEIR Models

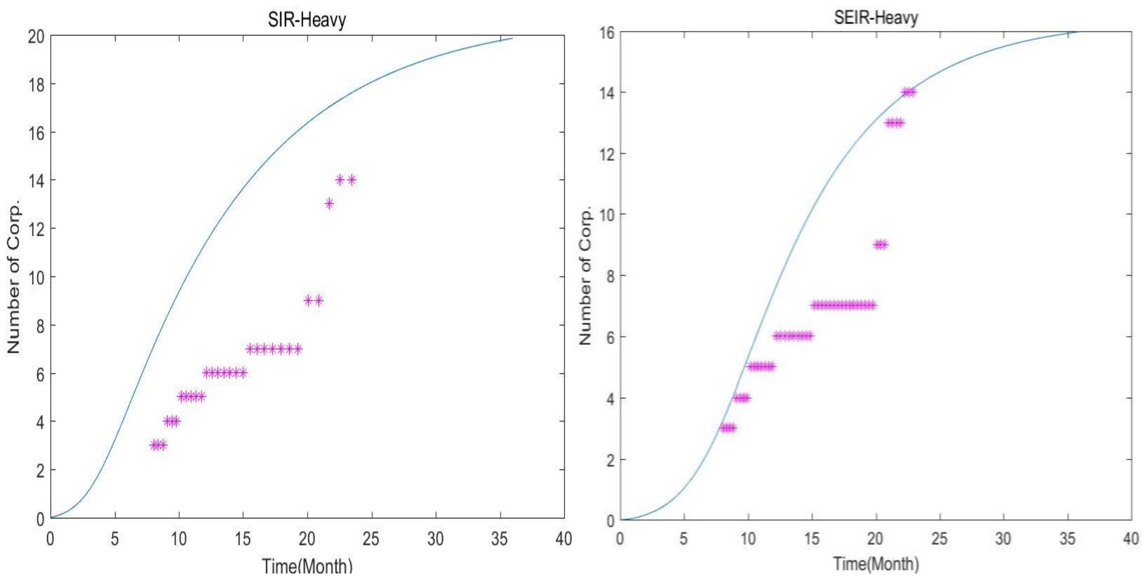


Among the total 555 companies, 23 companies have been delisted as of July 2010. At that point, the SIR simulation result is '30.3362688232418 'and the SEIR simulation result is '24.9161762146408'. Existing research has suggested that EDF model can be used as an early warning. (Jieun, H., 2009) Therefore, the result of SIR model can be interpreted as predicting the number of bankrupt companies four months in advance. However, even though we consider the suggestion of existing study, the error is very large. This implies that SIR model overestimates the number of bankrupt firms. On the other hand, the result of SEIR model is expected to predict the default of companies about one month ahead of bankruptcy, except for errors caused by concentration of the abolition in April and May 2010. In addition, the number of bankrupt companies that occur during the whole period can be predicted appropriately.

3.2.2. Simulations for Heavy Industry

[Figure 5] is the 'D (Dead)' state graph of the heavy industry simulation results using the ODE45 function of Matlab.

[Figure 5] Simulations for Heavy-Industry Corporations by SIR & SEIR Models

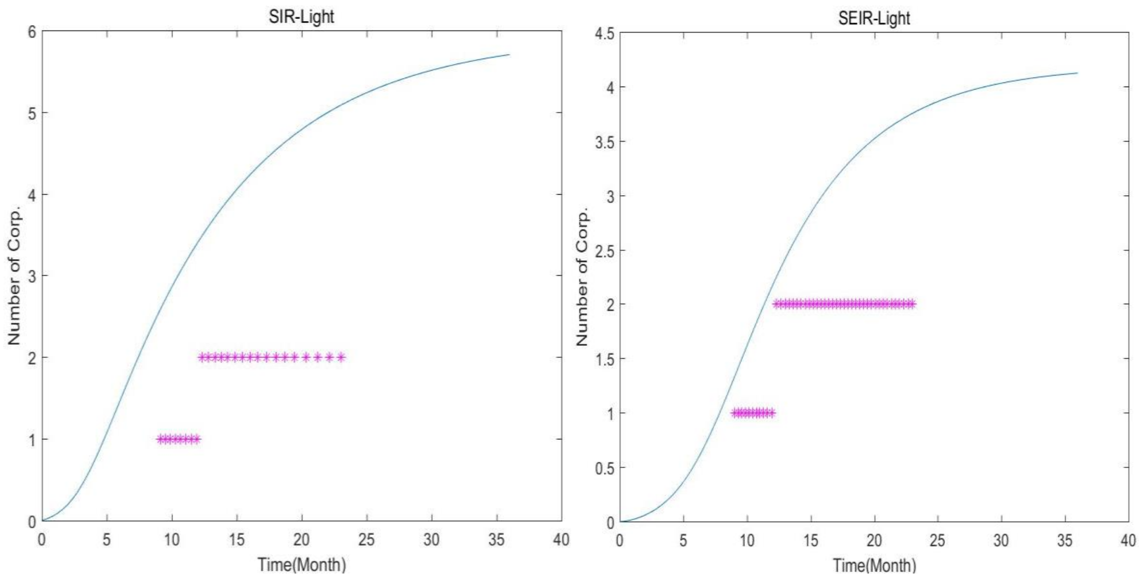


Among the 302 companies classified as heavy industry, 14 companies have been delisted as of July 2010. At this point, SIR simulation result is '17.3119098700968 'and SEIR simulation result is '14.1280403974764'. Since more than half of whole companies and of the delisted companies are classified as heavy industry, the result is similar to the result for whole companies. The result of SIR model can be interpreted as predicting the number of bankrupt firms three months ahead, but still overestimates the number of bankrupt firms because it has the steady error over 23 months (August 2008 - July 2010). The result of SEIR model shows that the number of bankrupt firms is more accurately predicted than that of the whole firms.

3.2.3. Simulations for Light Industry

[Figure 6] is the 'D (Dead)' state graph of the light industry simulation results using ODE45 function of Matlab.

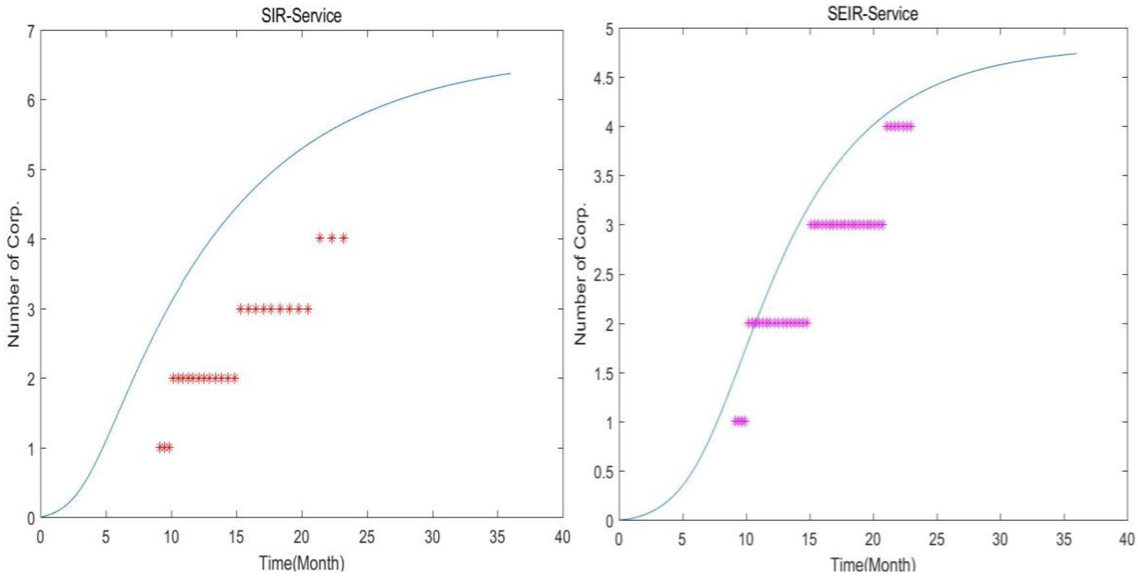
[Figure 6] Simulations for Light-Industry Corporations by SIR & SEIR Models



Since the last bankruptcy occurred in August 2009, it is necessary to check 'D' state after 12 months based on August 2008. Of the 89 companies classified as light industry, the total number of companies whose delisting has been confirmed as of August 2009 is 2. At this point, SIR simulation result is '3.489284887254144' and SEIR simulation result is '2.157797908875153'. As in the case of heavy industry, the result of the simulation using SEIR model compared to SIR model predicts the number of bankrupt firms very properly. Heavy industry and light industry can be bound to the manufacturing sector. Thus, the result using SEIR model is very meaningful in that it can accurately predict the domestic market which is highly dependent on the manufacturing industry.

3.2.4. Simulations for Service Industry

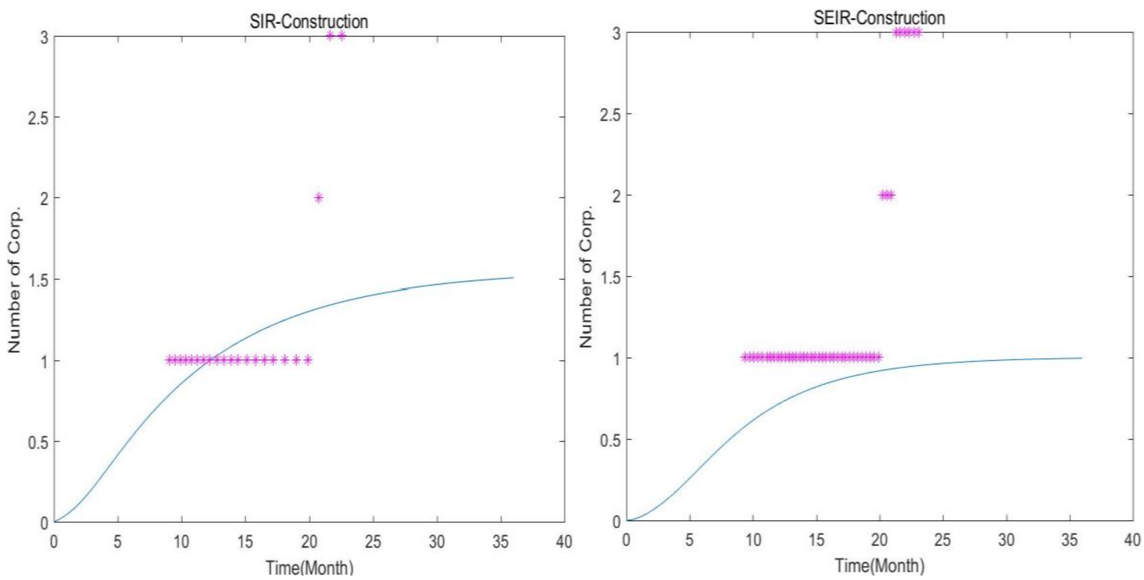
[Figure 7] Simulations for Service-Industry Corporations by SIR & SEIR models



Among the 100 companies classified as service industry, 4 companies have been delisted as of July 2010. At this point, SIR simulation result is '5.65461985663189' and SEIR simulation result is '4.28946524652111'. Similar to the previous results, SIR model results can be interpreted as overestimating the number of bankrupt firms, and SEIR model predicts the number of bankrupt firms significantly. The companies included in the service industry seem to have a clean financial structure because they are mainly composed of firms dealing with public goods such as gas supply companies and networking business, and holding companies of large corporations. Therefore, it is easy to predict bankruptcy using the epidemic model because accurate data were available.

3.2.5. Simulations for Construction Industry

[Figure 8] Simulations for Construction-Industry Corporations by SIR & SEIR models



Of the 19 companies classified as construction industry, the total number of companies whose delisting has been confirmed as of July 2010 is 3. At this point, SIR simulation result is '1.35993493019388' and SEIR simulation result is '0.940083673616135'. [Figure 8] shows that the results of SIR and SEIR model for the construction industry are very inaccurate. As results of researches about EDF model by classifying industries (Dongpil, S., 2010), (Jieun H., 2009), in case of construction industry, EDF value of the defaulting company declines rather immediately before default occurs. This is an error that occurs because the financial structure of the construction industry is forming an excessively high debt ratio. Therefore, in order to apply the KMV EDF model, it is necessary to study the calculation of the correct default point through the introduction of contingent liabilities or through the estimate appropriate adjustment factor α for DPT calculation. In this study, because the included construction firms have already formed high EDF value before the financial crisis, the accuracy of the parameters, determined by EDF changes before and after the financial crisis, is considered to be low.

IV. Conclusion

4.1. Conclusion

The purpose of this study is to validate the financial crisis diffusion analysis using the epidemic models (SIR model and SEIR model). Based on the results of KMV EDF model, which uses stock price information of publicly traded companies to predict defaults, the necessary parameters for the simulation are obtained. We want to verify whether the epidemic model can be used to predict actual bankruptcies for each industry classification.

The companies listed on the KOSPI market were classified as heavy industry, light industry, service industry, and construction industry. Simulation was conducted using the daily closing price, market cap data and monthly debt information from January 2007 to December 2010. The main results of this paper are as follows.

First, we analyzed the results of SIR and SEIR models for overall corporations, and found that SEIR model predicts the number of bankrupt firms significantly. In case of SEIR model which contains incubation period (latent period), ‘delay’ occurs in the transition to the 'R' and 'D' states after the infection. Thus, the bankruptcy occurs later in the simulation than in SIR model. This is related to the procedure until the enterprise's bankruptcy is decided. If a company fails to pay the amount due on the due date of the bill or check, the company will be in a bankruptcy crisis. However, in order for the bankruptcy to be confirmed, procedures such as ‘Composition’, ‘Corporate reorganization procedure’, and ‘Finance structure improvement agreement (workout)’ are necessary.[12] As a result, there is a ‘delay’ until the company reaches the final bankruptcy, and this delay is effectively reflected by the concept of ‘Latent period’ of SEIR model.

Second, the simulation using SEIR model can predict the number of bankrupt companies in heavy industry and light industry very accurately. This is the result of adding the significance of the analysis of the spread of the financial crisis using SEIR model in the domestic market where the manufacturing

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[12] Composition: Procedures for creditors and debtors to agree on how to repay bonds under the supervision of a public agency such as the court

Corporate reorganization procedure: Procedures for rebalancing the interests of stakeholders under the supervision of a court if they are financially bankrupt companies but are likely to be rehabilitated

Workout: Readjustment of debt by private agreement with domestic financial institutions industry is the main industry. Also, in the case of the service industry, it is found that the simulation using SEIR model predicts the number of bankrupt companies properly. In the case of the construction industry, it seems that the prediction of the default using the epidemic model is not appropriate, but it can be improved by expanding the scope to the KOSDAQ listed company and increasing the number of specimens and including contingent liabilities.

In conclusion, if KMV EDF model using the stock price information and SEIR model based on EDF are applied to the forecasting of bankrupt companies, it is expected that the bankruptcy prediction considering the volatility of the economic situation and the enterprise value will be possible. In addition, by predicting bankruptcy in advance, it will be possible to control the spread of the financial crisis by preventing the second loss of investors and financial institutions. Therefore, the analysis of the spread of the financial crisis as an infectious disease model could lead to more meaningful results.

4.2. Discussion

The limitations of this study are as follows.

First, it is impossible to know directly whether an individual company has failed. The results of this study can predict only the number of companies that are subject to default in the classified industry, so it does not show which company belongs to the high risk group. This is due to the fact that SIR and SEIR models are models for solving differential equations. Therefore, if we use mathematical modeling such as ABM (Agent Based Model) which can check the variation of each object, it will be possible to predict mathematically the process of financial crisis spreading to individual companies.

Second, accurate results could not be calculated because more than 100 listed companies whose debt and stock price information were not clear were excluded. In particular, since the debt information used in this paper was only available for quarterly data, we used a very inaccurate method called 'Linear interpolation' to make it monthly data. Moreover, quarterly debt data is also not readily available for KOSDAQ listed companies and SMEs. For future research, it is very important to have reliable debt information, but there are practical limitations.

The last limitation is that there is no reference or clues why an exposed corporation is confirmed if EDF value did not decrease to the previous value during 2 quarters. Until now, there have been no studies linking infectious disease models to default prediction models, so it was necessary to make arbitrary decisions when defining parameters. Since I thought that it would take at least two quarters before a company recognizes the crisis, it was based on the period of 2 quarters. However, a clear reference will be needed for more accurate analysis in the future.

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V. Appendix

[Table 10] Descriptive Statistics about EDF for Heavy-industry

Year	EDF			
	Average	Max	Min	STDEV
2008	0.0435742548902419	1.0000000000000000	8.70460141898157E-81	0.115440700788568
2009	0.0547101631898132	1.0000000000000000	1.41605241724078E-25	0.105468300358232
2010	0.0204077894184747	1.0000000000000000	1.61949209333242E-70	0.0976966420105495

[Table 11] Descriptive Statistics about EDF for Light-industry

Year	EDF			
	Average	Max	Min	STDEV
2008	0.0246738888539544	0.421207299112572	6.70362373181453E-29	0.056575565053979
2009	0.0338913542106953	0.596021242560492	9.35913430398396E-56	0.0595217353872954
2010	0.0177706722454022	0.811579963758908	3.21987987701141E-71	0.0634043282828762

[Table 12] Descriptive Statistics about EDF for Service-industry

Year	EDF			
	Average	Max	Min	STDEV
2008	0.0126182994741962	0.475133501343474	2.50930021628462E-56	0.0364808156577184
2009	0.0256270815999402	0.998186439415881	1.20414836409626E-34	0.078155548277743
2010	0.028568721197529	0.998132103193509	9.54905066032942E-56	0.141467234185709

[Table 13] Descriptive Statistics about EDF for Construction-industry

Year	**EDF**			
	Average	**Max**	**Min**	**STDEV**
2008	0.0812288458206688	0.409628671399023	0.000117454523374174	0.0911573887214347
2009	0.134627695664919	0.669998857381605	0.000167255438365504	0.121283486584195
2010	0.0342455218292105	0.975865689456171	0.0000133430138941481	0.116706358188002